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Congestable Local Public Goods in an Urban Setting

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In this paper, we recognize two types of unpure local public goods (LPGs): congestable and pollutable. This paper and the forthcoming one (Clubs in an urban setting, mimeo, 1980) are devoted to congestable LPGs (CLPG). In Part I we define, classify, and characterize CLPGs. CLPGs may vary in their degree of congestability between zero, in the case of a pure LPG, and one, in the case of a private good. Spatially, we recognize two types of LPGs: the dispersed LPG (DiLPG) and the concentrated LPG (CoLPG). The first is distributed throughout the residential ring and the second is provided in specific locations to which households must travel. The second part of this paper is devoted to the investigation of the properties of a CDiLPG in an urban setting. The properties of CoLPG are investigated in (Clubs in an urban setting, mimeo, 1980). A version of Samuelson's rule as to the optimal allocation of pure public goods is extended on one hand to local dispersed public goods and on the other hand to CLPGs. Two corrective Pigouvian taxes are identified: congestion tolls levied on households, and a residential land tax. These two taxes cover total government expenditure on the LPG. The fraction of the expenses on the LPG covered by congestion tolls is α, and that covered by land taxes is $(1 - \alpha)$, where α is the degree of congestability. Efficiency can also be achieved by zoning the residential ring and regulating the housing density in it. This procedure may replace taxation and its advantage is that we can control a large number of externalities by a single set of zoning rules and regulations. An a posteriori rule to guide the local government as to the desirability of its actions is provided.

Recently an abundance of papers have appeared in the literature on the subject of local public goods and environmental externalities. However, no serious attempt has been made to classify the different types of local public goods or externalities and characterize their different properties. In this paper, an attempt is made to classify local public goods (LPG). We begin by recognizing two types of local externalities—congestion and pollution, and two types of LPGs—congestable LPGs (CLPGs) and polluted LPGs (PLPGs). This paper and the forthcoming one (Hochman [12]) are devoted to the investigation of CLPGs, and their characteristics in an urban environment.

Another possible classification of LPG is between dispersed LPG and concentrated LPG. In this paper we investigate the production, consumption, and provisional properties of a dispersed congestable LPG (DiCLPG)

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and its spatial characteristics. In our forthcoming paper [12], we investigate the properties of a congestable concentrated LPG (CCoLPG).

The main results of this paper are;

(1) Congestable local public goods can be ranked with respect to their degree of congestability $\alpha, 0 \leq \alpha \leq 1$. In the extreme case in which $\alpha = 0$, the CLPG is a pure public good and when $\alpha = 1$ the CLPG becomes a private good (the good can be provided by private markets) provided collectively.

(2) To achieve Pareto optimality the local government must provide at each location a quantity of dispersed congestable local public good which equates the rate of product transformation (RPT) between the public and the private good to the rate of consumption substitution (RCS) between the public and the private good multiplied by the density of population at the power of $(1 - \alpha)$, where α is the degree of congestability. This is an extension of the well-known Samuelson's rule concerning the optimal provision of public goods, to the case of congestable public goods on the one hand, and to dispersed public goods on the other.

(3) In order to achieve optimal spatial distribution of users of the LPG when the public good is not pure (i.e., $\alpha \neq 0$), congestion tolls which vary with location are to be levied on households.

(4) When a dispersed LPG is concerned a Pigouvian corrective tax has to be levied on residential land in order to avoid the residential ring from extending to the CBD on the one hand and the agricultural ring on the other hand.

(5) The optimal congestion tolls together with the Pigouvian taxes on residential land are equal to the cost of the government of providing the optimal level of the CDiLPG. When the degree of congestability is α, congestion tolls cover the αest part of the costs and taxes on land the $(1 - \alpha)$est part. When $\alpha = 1$, congestion tolls are equivalent to prices and by themselves cover the total cost. Thus when $\alpha = 1$ the DiCLPG has the nature of a private good.

(6) There is an a posteriori rule by which the local government can determine whether or not its actions are in the right direction: If total land rents increased by more than the total increase in the government's expenditure on the LPG, then the government action in providing the LPG contributed to efficiency. If the increase in land rents was exceeded by the increase in the government's expenditures then the government overshot its mark and invested too much in the LPG.

(7) By regulating the number of households per unit land in residential zones internalization of many of the external congestion effects can be achieved without the need for many complex calculations of congestion tolls. Therefore, this procedure is often superior to taxation.

PART I: THE CONCEPT OF A LOCAL PUBLIC GOOD

1. *Local Externalities and Local Public Goods*

DEFINITION 1. An externality caused by an activity in one type of land use which affects an economic activity in another type of land use is defined as a pollution type externality.

An externality caused by an activity in one type of land use which affects participants in the same activity is defined to be a congestion type externality.

Since all economic activity in an urban area uses land, the above two types of externalities encompass all local external effects.[1]

The number of local external effects is considerable. Pollution type externalities include air, land and water pollution effects that industry has on residential areas and/or other industries as well as traffic pollution affecting residential and other land uses. The effect high risers have on low rise buildings is also a pollution effect as is the effect a tree planted by a household too close to its lot limits has on the welfare of its neighbors. Among congestion effects we find traffic congestion, congestion of public transportation facilities, of schools, of parks, of jails and of all types of local public facilities. The effect of an industry's pollution on itself is also a congestion effect.

DEFINITION 2. A local public good (LPG) is defined to be a public good available only to users located in a specific location.

An LPG can be either pure or impure. In reality, there are very few, if any, pure local public goods, but the concept is useful as a starting point. An impure public good may be considered to be a combination of a pure public good with an external effect. Since there are two possible types of local externalities we will recognize two types of LPGs.

DEFINITION 3. A local public good subject to the external effect of congestion is defined as a congestable local public good (CLPG). A local public good subject to the external effect of pollution is defined as a pollutable local public good (PLPG). Since all impure LPGs can be described as pure LPGs subject to an externality, these two types of LPGs exhaust the entire range of possible LPGs.

Of course, in practice a public good can be both pollutable and congestable; however, clarity of the analysis demands separation of the issues. This paper is devoted to the analysis of congestable local public goods.

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[1] By a local externality it is meant that both, the cause and the effect of the externality are within the city boundaries. Since an externality is caused by a land use and affects a land use our statement holds. Externalities that are not local like the noise of a passing plane, radioactive fallout, or sewer discharge in the deep sea, can be treated as spillover effects. For a discussion of such an effect see [13].

2. CONGESTABLE LOCAL PUBLIC GOODS (CLPG)

When discussing the consumption of a desirable local public good we usually have in mind services provided by the LPG to the public, at a given location. The term services is used to indicate that the act of consumption of the LPG does not affect its capability to provide more of the same services, which is what makes it a public good. Examples of common LPGs are roads, parks, lakes, police protection, sewers, etc. It therefore seems useful to refer to a public good as having a given capacity which is instrumental in the provision of services to the consumer. We thus may refer to the width of a road as a measure of its capacity which is instrumental in providing travel services. The size of a park is a measure of its capacity which is instrumental in the provision of recreation services and the number of hours a television station broadcasts is a measure of its capacity to provide this sort of entertainment to its viewers, etc.

In this paper, the capacity of the LPG is also used as a measure of the quantity of the LPG. By doing this we implicitly assume that conversion of quantity to capacity is done via a process with constant returns to scale. This, of course, is a simplifying assumption. In most real world cases there are economies or diseconomies of scale involved in this process. For example, when the width of a road is doubled the capacity is more than doubled. However, since the focus of our study is not on the production side of the LPG, these aspects are rather unnecessary complications and are therefore assumed away.

The amount of services consumed by a household depends not only upon the capacity of the public good but also on the utilization of it by the household. Thus, the utilization of a road providing travel between A and B to a household depends also on the amount of time spent on travel by the household between those two points, which in turn depends on the speed and the type of vehicle the household uses. Recreational services obtained by a household from a park depend on the amount of time spent by the household in the park. The amount of entertainment from a TV station depends on the amount of time spent by the household watching it, the type of TV set the household possesses, etc. The measure of utilization may or may not be in the same units in which the capacity of the LPG is measured.

The total or relative amount of utilization of the LPG by all the users may also affect the amount of services available to an individual household. When such an effect exists we say the public good is subject to congestion.

DEFINITION 4. We say that a household i derives services, q_i, from a CLPG with capacity C, level of utilization of the CLPG by the household μ_i, and congestion factor facing the household k_i—when there is a function ψ^i, so that

$$q_i = \psi^i(C, \mu_i, k_i), \qquad \partial\psi^i/\partial C > 0; \ \partial\psi^i/\partial\mu_i > 0; \qquad \partial\psi^i/\partial k_i < 0, \tag{1}$$

and

$$\begin{aligned} &k_i = k_i(\mu_j;\ j \in J), \\ &\partial k_i/\partial \mu_j > 0 \qquad \text{for some } j \neq i, \\ &\partial k_i/\partial \mu_j \geq 0 \qquad \text{for all: } j \neq i. \end{aligned} \tag{2}$$

Where J is an index set of all households using the same CLPG and $i \in J$.

Thus the travel from point A to point B on a given public road is a function of the capacity C of the road, the value of the time the traveler spends on the road together with the cost of gasoline and the amortization of the car during the trip, all of which constitute the level of utilization μ_i of the household. The congestion factor in this case is the number of vehicles on the road at the time the household is traveling.

We recognize two major types of congestion factors. One is the relative utilization of the public good measured by the utilization ratio given by (2.1). A similar concept of a congestion factor is used by Oakland [19]. From (2.1) it follows that when all households utilize the LPG equally, k_i equals the number of households using the capacity. However, when some households utilize the public good more intensively than others, the amount of services available to others is decreased.

$$k_i = \left(\sum_{j \in J} \mu_j\right)/\mu_i. \tag{2.1}$$

The alternative form of a congestion factor is the total utilization given by (2.2):

$$k_i = \sum_{i \in J} \mu_j. \tag{2.2}$$

In this case the congestion factor affects all households similarly. The appropriate factor to use depends on the public good in question. For example, if the LPG is police protection then the utilization of the LPG depends not only on its capacity, the number of policemen on patrol, but also on protective measures taken by the household, such as locks on doors, fence around the house, bars in the windows, burglar alarms, and even the possession of a whistle, all of which constitute the utilization level. If a household installs more sensitive alarm devices than his neighbor, it is likely to draw the attention of the local patrolmen more effectively than others in a case of a possible burglary and thus reduce the protection provided to the rest of the area. If, however, all neighbors install the same devices and all use the same utilization level, all will receive the same amount of protection. Therefore, it appears that the utilization ratio is the appropriate congestion factor to use in this case.

When dealing with parks, for example, a single household staying twice the time of another household will contribute twice as much to congestion. In this case, therefore, total utilization of the LPG by all users seems to be the appropriate congestion factor.

The function $\psi(\cdot)$ may be considered a household production function in the Becker–Lancaster context. We *do not*, however, need the strong assumptions of linear homogeneity of the function used by Becker.

3. *The Degree of Congestability of an LPG*

A public good with capacity C is considered to be a pure LPG if

$$\psi^i(C, \mu_i, k_i) = \psi^i(C, \mu_i, 1) \tag{3.1}$$

for all $k_i > 0$. A good is considered to be a private goods when

$$\psi^i(C, \mu_i, k_i) = \psi^i(C/k_i, \mu_i, 1). \tag{3.2}$$

It is apparent that in the case of (3.1) the total or relative amount of utilization of the good does not affect the level of services to the users; that is, each user utilizes the total amount of the good. In the case described by (3.2), however, we see that each user gets services from only that portion of the total capacity which is proportional to his share in the exploitation of the good measured by k_i. In other words, (3.2) describes a case of a private good (see also H–P–B [7]). Indeed, due to technical or other considerations, the good may be provided by the public although it is actually a private good. It is interesting to note in passing that city roads subject to congestion are such private goods that technology dictates should be provided collectively. To see this, consider the congestion function given in Mohring[2] [17, p. 16], which states that if T is the travel time per trip, N is the number of cars on the road on which the trip is made and K is the road's capacity, then $T = f(N/K)$. In our terminology K is C, the capacity of the road which is the public good, μ_i the utilization level is measured in units of time spent on the road, N is the congestion factor k_i, and q_i is the number of trips made by the household. Thus,

$$q_i = \frac{\mu_i}{T} = \frac{\mu_i}{f(N/K)} = \frac{\mu_i}{f\left(\frac{k_i}{C}\right)} = \psi^i(C, \mu_i, k_i). \tag{3.3}$$

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[2] See also Vickery [25] and others.

It is easy to see that ψ^i in (3.3) indeed fulfills (3.2), which makes urban roads private goods collectively provided.

The implications of such a relationship will be discussed later in this paper and the specific results for the particular example of congested roads is discussed in detail in Hochman [8, 9]. One such result is that optimal congestion tolls exactly cover expenditure on the road and no additional sources of financing are required. Thus congestions tolls serve as prices.

DEFINITION 5. A good with capacity C, utilization level μ and congestion factor k is said to be a CLPG of degree α, $0 \le \alpha \le 1$ when there is a function $\lambda(k)$ such as

$$\psi(C, \mu, k) = \psi(C/\lambda(k), \mu, 1),$$
$$\alpha = d\ln\lambda(k)/d\ln k. \tag{4}$$

It follows that when $\alpha = 1$, the LPG is a private good and when $\alpha = 0$, it is a pure LPG. When $0 < \alpha < 1$ the whole range of CLPGs[3] is covered.

The degree of congestability of a public good, α, defined in (4) as an elasticity, can in general vary as a function of k, c, and even μ. For reasons of convenience and practicality we limit discussion to the case in which α is constant, i.e.,

$$\lambda(k) = k^{\alpha}, \tag{4a}$$

which implies

$$\psi(C, \mu, k) = \psi\left(\frac{C}{k^{\alpha}}, \mu, 1\right). \tag{4b}$$

It is likely that in practice this assumption will suffice for relevant ranges of congestability; however, it would be necessary to confirm it empirically for any particular case. It has already been proven to be the case for congestable roads (see Vickery [25] and Mohring [17]).

Equation (5) defines the function $q(C, \mu)$:

$$Q(C, \mu) \overset{\text{def}}{=} \psi(C, \mu, 1). \tag{5}$$

The i's household production function of services q^i from a CLPG of degree α is now given by

$$Q^i\left(\frac{c}{(k_i)^{\alpha}}, \mu_i\right), \qquad Q_1^i > 0, Q_2^i > 0. \tag{6}$$

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[3] When $\alpha > 1$ it is implied that there are diseconomies of scale of the utilization ratio. This kind of good should be called a congested private good and actually belongs to the domain of external effects between consumers. The following mathematical derivations hold for this case as well, although not necessarily with respect to their rational.

5. *Classification of LPG According to Spatial Aspects*

From the spatial point of view we may distinguish between two types of LPG. One type, which may be termed dispersed LPG (DiLPG), is consumed at the dwelling place of the household and thus has to be distributed between those dwelling places. Examples of DiLPGs are police protection, municipal sanitary services (garbage collection, street cleaning, sewers), TV and radio broadcasts, etc. The level of utilization of police protection, for instance, depends also on protective measures taken by the household. The utilization of municipal sanitary services also depends upon the existence of appropriate facilities in the house like toilets, garbage cans, etc. Note that in the above examples the household uses very little or none of its own time. However, in the consumption of TV and radio programs, the household, besides investing in TV, radio sets, and antennas, must invest its own time as well.

The other type of LPG, which will be termed concentrated LPG (CoLPG), is available to consumers only in specific locations to which households must travel to make use of the services. Examples of CoLPG are schools, parks, public swimming pools, museums, theater halls, etc. What characterizes these goods is that, beside travel time, households also have to spend time in utilizing the services.

It is interesting to note that some of these goods such as schools, theaters and swimming pools are provided both publicly and privately. The majority of the CoLPGs, however, are CLPGs of varying degrees.

We conclude the above discussion with the following definition:

DEFINITION 6. A DiLPG is a local public good whose services are consumed by residents in their dwelling place. A CoLPG is a LPG located in a specific location to which the household must travel in order to consume it.

It seems logical that in the case of a DiLPG the appropriate congestion factor to use is the relative utilization (see (2.1)), as in the case of police protection. When a CoLPG is considered, the appropriate congestion factor seems to be the total utilization because, in this case, utilization involves investment of the household's time. Accordingly, in what follows, k_i, the congestion factor is assumed to be the utilization ratio when a DiLPG is involved and the total utilization level when a CoLPG is discussed. However, although this sorting seems to be appropriate when considering the examples, cases in which the situation is different are possible and results of this and the author's forthcoming paper [12] still hold for them, perhaps with slight modifications.

Public transportation facilities, like roads, bus, and train services seem to fit into the category of DiLPGs even though the population they are intended to serve is the population of travelers rather than the residential population. In other words, this type of LPG is a category of its own. Public transportation has been discussed thoroughly in the literature (see, for example, contributions by Mohring and Strotz). Locational aspects of city roads have also been discussed in recent papers of the author (see [8, 9]) as well as others. Therefore, we will not discuss it further in this paper.

Even when spatial and commuting problems are ignored, the existence of a CLPG will lead to the formation of clusters of households, each cluster utilizing the LPG in its proximity. The optimal size of a cluster is that size where the cost of an additional household to society due to congestion exactly matches the benefits to the marginal household from the consumption of the LPG. This fact led to the development of the theory of clubs, originated by Buchanen [4], and followed up by contributions by Berglas [2] and others. It is obvious that when $\alpha = 0$, and no travel costs exist, only one club will form, i.e., the whole nation. When $\alpha = 1$ each household will consume its own LPG and thus be a club of a single household. When α varies between zero and one the number and sizes of such clubs varies between those two extremes.

When taking into account spatial aspects, even in the case of a pure LPG, clustering will occur due to limited accessibility to the LPG. This concept of a LPG is probably the one originally visualized by Tiebout [23]. We will prove that some of the simple results of club theorists do not hold when spatial aspects are taken into account. One of these results which do not hold is that congestion tolls by themselves are always sufficient to finance the LPG. As an example, when $\alpha = 0$, no congestion tolls exist. Another result, proven below, is that the rules for optimal club size should be altered. Optimal provision of CoLPG often leads to the creation of clubs. This and the spatial aspects of clubs and CoLPGs are discussed in [12].

PART II: MODELING DiLPGS

1. *Assumptions and Notations of the Model*

Consider a city with an industry located in the central business district (CBD) in the center of town. The residents of the city live in the residential ring spread around the CBD and are employed in the CBD. They belong to a homogeneous population group which exists throughout the economy and which consists of households having the same utility function, identical skills in the labor market, and owning property which yields the same income, independently of the household. The utility levels of the members of the population group are equal to each other, and do not depend on the city of residency, or location within the city. If the utility level was not identical, members of the group would migrate to locations where utility is higher, bid up the land rents in the desirable locations, bid it down in the less desirable areas and thus cause utility to equalize throughout the economy. These are by now standard assumptions in the literature. For further details and implications see Hochman [9, 10, 13].

The following is a list of parameters, functions, and variables (parametric and functional) used in the model.

Parameters

x —The distance from the center of the city.
U_0 —The given utility level of the population group residing in the city.
V —The exogenously determined level of nonearned income of the population group.
R_A—The agricultural rent in areas adjacent to the city.
α —The given degree of congestability of the DiLPG.

Functions

$U(h, Z, q)$— Utility function of the population, where h is quantity of housing, q are services obtained by the household from the public good, and Z is the quantity of all other goods.
$t(x)$— Commuting costs per household from x to the center of the city.
$F(\bar{c}, G, N)$— Transformation curve between $\bar{c}$, the total quantity of the public good produced in the CBD and the value of the total city export of the good produced there. G and N are production factors.
$A(x)$— $A(x)dx$ is the total quantity of land in a concentric ring with a radius of x and width dx.
$Q(C/k^{\alpha}, \mu)$— The household production function of services from a DiLPG with capacity C, utilization level μ, congestion factor k, and degree of congestability α.
$k(x)$— Congestion factor, assumed in our model to equal the relative utilization.

Variables
Parametric Variables

N— Total amount of the population and labor force of the city.
G— Total quantity of land in the CBD.
$\bar{c}$— Total quantity of the public good produced by the industry in the CBD.
D— The CBD limit
L— The city limit

W—Wage rate paid in the center of the city.
a—Land toll per unit of residential land adjacent to the CBD.
b—Land toll per unit of residential land at the city limit.
g—Price per unit of the LPG paid by the government to the industry producing the LPG.

Functional Variables

The following are all functions of x. In what follows we shall avoid using the variable x unless clarity of exposition requires it.

h—Quantity of housing consumed by the household, measured in units of land.
q—Quantity of services of the public good consumed by the household.
C—Capacity of the public good per unit land in x.
μ—Amount of utilization of the public good by the household, measured in units of income.
R—land rent.
T—Congestion toll levied on households.
e—Taxes on land rents.

We assume that the local government is not involved in production and buys the public good from private industry at a market price, g, and then distributes the good to the local population. Thus, the industry in the CBD is involved in producing two goods. One, $\bar{c}$, is sold to the local government; the other is the city's export good which is sold for a fixed price. Let $F(\bar{c}, G, N)$ be the value of the private export good produced by the industry if a quantity $\bar{c}$ of the DiLPG is also produced by G land and N labor.

$$F(\bar{c}, G, N) \geq 0, \qquad F_C < 0, F_G > 0, F_N > 0, \tag{7}$$

where

$$G = \int_0^D A(x)\, dx \tag{8}$$

is the total land in the CBD.

The function $F(\cdot)$ may be linear homogeneous or it may be of decreasing returns to scale.[4]

The local government distributes the public good it purchases from the industry to the entire residential area, so that at each location the density of

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[4] For the formation of a city we need the existence of economies of scale in the export good industry. To avoid problems irrelevant to the issue at hand we do not pursue this subject here. For further details on the subject of economies of scale see [10].

the DiLPG is $C(x)$. Thus,

$$\bar{c} = \int_D^L C(x)A(x)\,dx. \tag{9}$$

The standard simplifying assumption that housing is produced by land only is used here as well. If $h(x)$ is the quantity of housing consumed by the household at x, then the density of households per unit land is $1/h(x)$. the total number of households, N, is therefore given by

$$N = \int_D^L [A(x)/h(x)]\,dx. \tag{10}$$

Note that N is also the total labor force of the city employed in the CBD. This follows from the assumptions that each household contributes one worker to the labor force and that the total labor force is employed in the CBD.

Each household spends $\mu(x)$ income units on utilizing the DiLPG. Thus, the total utilization level per unit land is $\mu(x)/h(x)$. The household's relative utilization is therefore

$$k(x) = \mu/[h \cdot \mu] = 1/h(x) \tag{11}$$

and is equal to the population density.

Substituting the above into (6) we find that the level of services produced to a household at location x from a public good with density of capacity $C(x)$ are

$$q(x) = Q\big(C(x)h(x)^{\alpha}, \mu(x)\big). \tag{12}$$

Assume that the government operates on a balanced budget constraint, and that taxes on land do not exceed land rents. They may, however, be negative; i.e., the government may choose to transfer excess income to land owners. Thus

$$e(x) \le R(x), \qquad 0 \le x \le L \tag{13}$$

2. *Equilibrium Solutions*

Define a decentralized allocation of resources in an economy to be an allocation in which the government's actions are restricted to taxing, subsidizing or redistributing goods or income and all other economic activities are carried out in competitive markets. The laissez-faire equilibrium is a decentralized equilibrium allocation in which the government does absolutely nothing.

The following equations describe a decentralized equilibrium allocation in a city with a local government in accordance with the assumptions and notations of the previous section.

(14)	$U(h(x), Z(x), q(x)) = U_0$	The constant utility constraint.
(15)	$-F_C = g$	Equality of rate of products transformation to price ratio.
(16)	$F_G = R(D) - a$	Equality of value of marginal productivity of land to the residential land rent minus a residential land tax, a.
(17)	$F_N = W$	Equality of value of marginal productivity of labor to the wage rate in the CBD.
(18)	$R(x)h(x) + Z(x) + \mu(x) + t(x) + T(x) = W + V$	The budget constraint.
(19)	$R(x) = U_h/U_Z$	Equality between the marginal rate of substitution of housing and the private consumption good, and their price ratio.
(20)	$1 = q_u U_q/U_Z$	Equality between the marginal rate of substitution of the utilization level and the private consumption good, and their price ratio.
(21)	$R(L) - b = R_A$	Equality of residential land rent at the city limit minus taxes per unit of residential land there, to the agricultural land rent.

These conditions represent an entire set of possible equilibria, each of which is determined by the local government's policy concerning a, b, $T(x)$, $e(x)$ and $C(x)$. (The quantity $\bar{c}$ is then determined through (9) and g can be solved from (15).)

The laissez-faire equilibrium is the one in which $a = b = T(x) = e(x) = C(x) = \bar{c} = 0$.

3. *The Locally Efficient Solution*

Let us define the surplus of the city, S, to be the value of the net exports of the city, i.e., the total value of export goods produced in the city plus the total amount of unearned income minus the value of all imports of consumption goods, services and other consumption inputs (such as alternative value of land). The city is a price taker to all exports and imports, including labor for which it pays in utility level.

$$S = F(\bar{c}, N, D) + NV - \int_D^L \frac{A(x)}{h(x)} dx \left(z(x) + \mu(x) + t(x)\right)$$
$$- \int_0^L A(x) R_A \, dx. \tag{22}$$

We say that a city is locally efficient if S, defined in (22), is at its maximum value subject to the constraints (8), (9), (10), (12) and (14). Local efficiency is a *necessary condition* for Pareto optimum in the entire economy. To see that, suppose S does not attain its maximum value in all cities in the economy. By increasing S, subject to the constraints, additional income is created which can be given to a household in the economy and thus increase its utility beyond the given utility level. This means that when S is not at its optimum we can make someone better off without making anyone else worse off. Therefore, when S is not at its maximum the economy cannot be in a state of Pareto optimum. Consequently, maximization of S is a necessary condition for Pareto optimum in the economy.

The surplus of a city of zero size, i.e., $D = L = N = 0$, is equal to zero. Since this solution is included in the feasible set of solutions it implies that the maximum value of S, S^* fulfills

$$S^* \geq 0. \tag{23}$$

For further details on local efficiency and Pareto optimum see [10]. The incentive of local governments to achieve efficiency is discussed in [11].

The decentralized solution described in the previous section will yield the optimum solution under the following values of the government controls.

$$g = (h(x))^{\alpha - 1} \left(u_q(x)/U_z(x)\right) q_c(x), \tag{24}$$

$$T(x) = \alpha (h(x))^{\alpha} \left(U_q(x)/U_z(x)\right) q_c(x) C(x), \tag{25}$$

$$a = gC(D) - T(D)/h(D), \tag{26}$$

$$b = gC(L) - T(L)/h(L). \tag{27}$$

PROPOSITION 1.[5] *The optimal dispersion over space of a pure* DiLPG *is such that the capacity of the* DiLPG *in each location equates the sum of the marginal rates of substitution between the* DiLPG *and the private consumption*

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[5] The proof of Proposition 1 follows immediately upon substituting $\alpha = 0$ and Eq. (15) into Eq. (24) and noting that $(-F_C)$ equals the rate of products transformation between the public and the private goods in the CBD.

good, of all the households consuming the DiLPG *at this location, and the rate of product transformation of the same two goods.*

Proposition 1 is an extension of the well-known Samuelsonian rule of efficient allocation of pure public goods to include spatial aspects as well.

Suppose we regard all households consuming the same capacity as a single consumption unit whose marginal rate of substitution between the LPG and the private good equals the sum of the private rates of substitutions between the public and private good of all the individual households composing this ficticious unit. Proposition 1 then implies that the marginal rate of substitution between a DiLPG and a private good is equal in any two locations. But that is exactly Pareto efficiency rule which holds when two *private* consumption goods are involved.

On the other hand Samuelson's rule holds for the DiLPG at any given location, and not for the entire population as is the case in Samuelson's original case. Thus we see that spatial aspects by themselves already tend to make even a pure LPG more like a private good. Proposition 2 tells us that congestion has the same effect.

Proposition 2 extends Samuelson's efficiency rule to congestable public goods. Actually, it generalizes the rules for efficient allocation of private and public goods to a single general rule of which these two cases are the two extremes.

PROPOSITION 2. *When dealing with a homogeneous population, and relative utilization as a congestion factor of a* CLPG, *in order to achieve efficiency the ratio between the marginal rate of substitution between the* CLPG *and the private good, weighted by the number of households using the* CLPG *at the power of one minus the degree of congestability, should equal the rate of product transformation between those two goods.*[6]

It is easy to see that when $\alpha = 0$, i.e., a pure public good, we are back at Samuelson's efficiency rule for public goods and when $\alpha = 1$ we are back with the rule for efficiency when private goods are concerned.

PROPOSITION 3. *Congestion tolls equal to $T(x)$ as in* (25) *should be levied on a household at x in order to achieve local efficiency. The ratio between congestion tolls paid per unit land to the total value of the* DiLPG *in this area is fixed and equal to the degree of congestability. The following relation therefore holds.*[7]

$$T(x) = \alpha g C(x). \tag{28}$$

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[6] The proof of Proposition 2 again follows from (24) after substitution of (15) into it.

[7] Equation (26) shows the need for congestion tolls to achieve efficiency. By substituting (24) into (25) we obtain (28). Since total expenditure on DiLPG per unit land at x is $gC(x)$, it follows that the ratio between the two terms is equal to α.

PROPOSITION 4. *When a* DiLPG *is provided in a decentralized equilibrium of a city it creates an external effect in the land market. To internalize this external effect a residential land tax equal to the value of that part of the* DiLPG *which has not already been paid for by congestion tolls at each location should be levied.*

Designate the land taxes per unit land at x by $TL(x)$; then[8]

$$TL(x) = gC(x) - T(x)/h(x) = (1 - \alpha)gC(x). \tag{29}$$

The rationale behind Proposition 4 is straightforward. Since residents do not pay the full amount for services they obtain from the DiLPG, they are willing to spend this extra money on land and thus outbid the CBD industry and the agriculture industry. The externalities involved here is that residents do not take into account the fact that if they settle an area the local government will have to provide them with DiLPGs of which they pay only a part through congestion tolls. In order to internalize this effect we have to charge residents with the full value of services they obtain from the DiLPG, this time, by imposing a tax on land.

Proposition 3 contradicts findings of club theorist who say that congestion tolls are sufficient by themselves to finance the public good. When spatial aspects are taken into consideration we see that this is not so. Proposition 4 tells us that tolls should be levied on land as well, and thus contradicts earlier findings (see, for example, H–P–B [7]) that no land taxes are required.

Note that although in general taxes on land do not carry a deadweight loss (as opposed to head taxes, for example; see [10, 11]), in our present case these taxes are corrective Pigouvian taxes and should be very specific. A land owner should have to pay these taxes if and *only* if his land is used as residential land.

Under our assumption, these two types of taxes by themselves cover the entire government expenditure on the LPG and thus

$$e(x) = 0. \tag{30}$$

By substituting (28) into (18) and rearranging terms, the following form of the budget constraint is obtained:

$$(R + \alpha gC)h + z + t + \mu = W + V, \tag{31}$$

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[8] It is easily seen that if $TL(x)$ is as in (29), both (26) and (27) are fulfilled. If $TL(x) = 0$, (26) and (27) imply that the residential ring will expand with respect to the optimum. Since $gC(x)$ is the value of the public good per unit land at x and $T(x)/h(x)$ is the total congestion tolls paid by households located on a unit of land at x, $TL(x)$ is indeed the part of the value of the DiLPG not paid for by congestion tolls.

where the term $R + \alpha gc$ is the rent and congestion tolls payed by the household per unit land.

By first differentiating (31) and then substituting the total differential of (14) the following version of the spatial equilibrium condition is obtained:

$$(\dot{R} + \alpha g\dot{C})h + \dot{t} - gh\dot{C} = 0, \qquad (32)$$

where a dot above the variable indicates differentiation with respect to distance.

It is easy to see that when $\alpha = 1$, (32) is reduced to the well-known regular, spatial equilibrium condition (see Muth [18]).

From (32) we see that $\dot{R}$ can be positive only if $\dot{C} > \dot{t}/(1 - \alpha)gh)$. The sign of $\dot{C}$ depends on the parameters of the functions in the model, especially the utility function and $Q(C, h, \mu)$, the household production function of public good services and is beyond the scope of this paper. It should be noted, however, that stability of the solution implies a negative $\dot{R}$, at least in some finite range in the neighborhood of the city limit.

If $\dot{C}$ is negative, $\dot{R}$ is also negative and so is $\dot{R} + \alpha g\dot{C}$, the rate of change with distance of total payments per unit housing rented by the household. This does not imply that $\dot{q}$, the rate of change with distance of the level of services obtained by the household from the public good cannot be positive, since it depends also on the rate and ratio of utilization of the public good by the household. The practical problem of calculating the desirable values of policy variables is a topic of current research.[9] Proposition 7 contributes to the solution of this problem for local governments.

4. *Zoning Regulation*

Pigouvian taxes are not a very practical method of internalization when dealing with urban externalities. There are too many of them, most of which are connected with various LPGs and other activities of the household. The calculation of a single toll is quite complicated, let alone all of them. It is therefore quite useful if, instead of the complex taxes, a simple method can be found that is relatively easy to implement even when a large number of different externalities are involved. The following proposition tells us that zoning regulations are such a method.

PROPOSITION 5.[10] *Zoning regulations that yield the optimal allocation are the following: The government should restrict* CBD *activity between the center*

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[9] Recently studies dealing in processes intended to reveal the demand for public goods has been published [6]. The difficulty in determining Pigouvian tolls is mentioned in [22], among others. A demand revealing mechanism is used to determine optimal Pigouvian tolls and optimal quotas is discussed in [3].

[10] The proof of this proposition follows upon solving the equilibrium solution subject to the constraints of the zoning regulations. Equation (33) follows when we note that R_Z is equal to $U_h/U_Z +$ the shadow prices of the two constraints of the zoning regulations.

of town and the optimal value of the CBD *limit, D^*, calculated from the model in the previous section. In the same way* ***residential activities should be restricted to the area between D^* and L^*****, where L^* is the optimal value of the city limit. This type of zoning replaces the residential land taxes which are intended to hinder the extension of the residential area beyond its optimal value. To control congestion using zoning regulations, the following procedure must be followed: at distance x, the government should* ***permit construction only of houses of minimum size $h^*(x)$****, where $h^*(x)$ is the level of $h(x)$ in the optimum solution (or alternatively the government should restrict the number of households per unit land to be no more than $1/h^*(x)$) and in addition provide the optimal level of the* DiLPG *$C^*(x)$.*

Let $R_Z(x)$ be the land rent when using zoning regulations and $R(x)$ the land rent when Pigouvian tolls are used then

$$\begin{aligned} R_Z(x) &= R(x), \qquad 0 \le x \le D, \\ R_Z(x) &= R(x) + g^*C^*(x), \qquad D \le x \le L. \end{aligned} \tag{33}$$

The government can now finance its activities by imposing any land taxes which fulfill (13). Obviously (33) implies that those land rents can always cover government expenditure.

Proposition 5 still does not tell us in what way zoning regulations are superior to Pigouvian corrective taxes. Propositions 6 and 7 do this.

PROPOSITION 6. *When more than one* CLPG *with relative utilization as the congestion factor exist, then for each* CLPG, *a congestion toll and a corrective land tax must be calculated at each location.*

When using zoning regulations *only one set of borders between residential and nonresidential areas has to be set and only one restriction on minimum housing size is required.* This minimum housing size is given in (34) as $\bar{h}(x)$.

$$\bar{h}(x) = \max_i h_i^*(x), \qquad i = 1, 2, \ldots, \tag{34}$$

where $h_i^*(x)$ is the minimum housing size required by the ith CLPG.

PROPOSITION 7.[11] *An a posteriori criterion to judge the desirability of a local government's actions, both, in the provision of an* LPG *and in the control of externalities, is the following:*

If the government's action results in an increase in land rents which exceeds the government's expenditure connected with it, then the action is desirable

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[11] The proof of this proposition follows directly from the following relation:

$$S = \text{TUR} - \text{NGE}$$

and contributes to efficiency. If, however, the government's action results in an increase in land rents which is less than the government's investment in the project, then the project is not desirable. The optimum is achieved when, for all government projects, a marginal change in the project (control of externality or provision of public good) causes a change in land rents that exactly covers the government expenditure.

SUMMARY AND CONCLUSIONS

The first part of this paper is devoted to the characterization and classification of local public goods (LPGs). We recognize two types of unpure LPGs: congestable and pollutable. This paper is devoted to the investigation of the congestable LPG (CLPG).

Generally, we assume that this quantity of services from a public good depends on the physical quantity of the LPG, which we term the capacity of the LPG. However, we also assume that it depends on inputs from other resources invested by the household which we term the utilization level. An LPG is congestable if the services to a given household are reduced when the utilization level of the LPG by others increases.

We define a measure of congestability α, which may vary along the unit segment. A CLPG with $\alpha = 0$ is a pure LPG and a CLPG with $\alpha = 1$ is a private good.

We also define a congestion factor for each household which increases when the level of utilization by others increases. The larger the congestion factor the smaller the amount of services the household gets from the LPG for a given capacity and utilization level of the household itself.

We especially concentrate on two specific types of congestion factors that seem relevant: The relative utilization factor and the total utilization factor.

Spatially we categorize the LPG into two classes: the DiLPG and the CoLPG. DiLPGs are public goods provided to its users at a location where these users perform another economic activity, such as residing, working, etc. CoLPGs are public goods located at certain specific locations and

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where

TUR—Stands for total urban rents (i.e., total rents in the area of the city minus the alternative agricultural value of the land).

NGE—Net government expenditure which means gross government expenditure on the LPG less income from corrective taxes (subsidies are included as negative taxes).

This relation follows when we execute a series of substitutions of the decentralized equilibrium conditions into (22). A detailed description of the necessary steps can be found in [9, 13], and especially [10]. The statements of the proposition now follow since we are interested in maximization of S.

consumers must travel to this location and spend time there in order to consume the CoLPG.

In the second part of the paper a model of a circular city is developed, in which industry is located in the CBD and the residential ring is located around it. A DiLPG is provided to the city's population at their residential location.

The purpose of the model is to help in the formulation of rules as to the optimal local government policy which will bring about local efficiency. The main results are discussed in seven propositions in the main body of Part II and listed in the introduction to this paper. These results are concerned with:

An extension of the well-known Samuelsonian efficiency rule concerning a pure public good to, on one hand, the case of LPGs where spatial aspects are involved and, on the other hand, to the case of unpure public goods. We also specify the congestion tolls to be levied on households and identify a Pigouvian corrective tax to be levied on residential land. We show that those taxes are sufficient to cover the local government's expenditure on the LPG. We then argue that a method of internalizing the congestion external effects, which is superior to the Pigouvian taxes exists. This method is the zoning of the residential and industrial areas and regulating the household's housing size. Land rents then absorb the previous corrective taxes and the government can finance the LPG by a single land tax. The superiority of this method is due to the fact that this method is simple to implement when many types of CLPG are involved. An a posteriori criterion which may guide the government as to the efficiency of its actions is provided as well.

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